Temperature Profile in Steam Generator Tube Metal

For structural design considerations it is necessary to determine the temperature distribution in the tube metal of a U-tube steam generator. The primary side of the steam generator contains a hot saturated fluid within the tubes and a saturated working fluid to be heated is on the secondary side on the outside of the tubes. The many tubes are arranged in a regular pattern to form a large tube bundle.

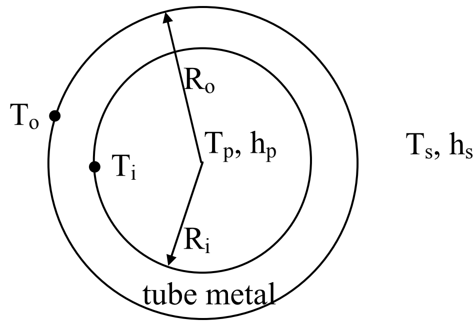

Fig. 1 Cross-sectional view of single steam generator tube.

Considering a single tube, the geometry becomes a simple circular annulus with the primary fluid on the inside and the secondary fluid on the outside as shown in Fig. 1. Here R_i, T_p, and h_p are the inside radius, primary fluid temperature, and heat transfer coefficient on the primary side, respectively. Similar variables, R_o, T_s, and h_s, define conditions on the secondary side. T_i and T_o are the inside and outside surface temperatures of the tube metal.

Radial energy transport through a tube wall with no internal heat generation is described by the heat conduction equation,

$$\vec{\nabla} \cdot \left(-k_m \vec{\nabla} T \right) = 0 \tag{1}$$

which, for constant thermal conductivity of the metal, k_m, and 1-D radial geometry, becomes

$$\frac{d^2T(r)}{dr^2} + \frac{1}{r}\frac{dT(r)}{dr} = 0 \tag{2}$$

Since both the inside and outside surfaces are exposed to convective environments, the boundary conditions become

$$-k_m \left.\frac{dT(r)}{dr}\right|_{r=R_i} = h_p\left(T_p - T_i\right) \qquad \text{(inside surface)} \tag{3}$$

$$-k_m \left.\frac{dT(r)}{dr}\right|_{r=R_o} = h_s\left(T_o - T_s\right) \qquad \text{(outside surface)} \tag{4}$$

where T_i and T_o refer to the metal surface temperatures and T_p and T_s are the bulk fluid temperatures. Equations (2) - (4) define a particular boundary value problem (BVP), giving T(r) within the tube metal as a function of the geometry (R_i and R_o) and the inside and outside fluid environments (T_p, h_p and T_s, h_s, respectively). Also, because of the convective boundary conditions, the tube metal surface temperatures must be determined as part of the overall solution procedure.

The primary and secondary side fluid environments are usually known and the heat transfer coefficients can be expressed in terms of the fluid environments. In particular, if we restrict our analysis to the single-phase turbulent flow region in the steam generator, then the desired heat transfer coefficients can be determined from various well-known heat transfer (HT) correlations.

First we define a number of dimensionless parameters:

Nusselt Number $$\mathrm{Nu} = \frac{h D_h}{k} \tag{5}$$

Reynolds Number $$\mathrm{Re} = \frac{\rho V D_e}{\mu} \tag{6}$$

Prandtl Number $$\mathrm{Pr} = \frac{\mu c_p}{k} \qquad (6)$$

where

$$D_h = \frac{4 \text{ x flow area}}{\text{heated perimeter}} = \frac{4A_f}{P_h} \qquad \text{(heated diameter)} \tag{7}$$

$$D_e = \frac{4 \text{ x flow area}}{\text{wetted perimeter}} = \frac{4A_f}{P_w} \qquad \text{(wetted diameter)} \tag{8}$$

$$\dot{m} = \rho A_f V \qquad \text{(mass flow rate)} \tag{9}$$

with the average fluid velocity denoted by V, and ρ, μ, k, and c_p are the standard symbols for the density, viscosity, thermal conductivity, and specific heat of the fluid, respectively.

Dittus-Boelter Correlation

One common HT correlation used for single phase applications is the Dittus-Boelter equation. On the primary side where the fluid is being cooled, the Dittus-Boelter correlation is

$$\mathrm{Nu}_\infty = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.3} \qquad \text{(for cooling of fluid)} \tag{10}$$

This expression is valid for turbulent flow inside tubes for 0.7 < Pr < 160 and Re > 10000. The ∞ symbol implies fully developed flow well away from any entrance effects.

On the secondary side where the fluid is heated, the Dittus-Boelter correlation is somewhat different. First, for heating of a fluid within a tube, the correlation becomes

$$\left(\mathrm{Nu}_\infty\right)_{c.t.} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.4} \qquad \text{(for heating of fluid)} \tag{11}$$

Also for flow along the outside of tube bundles, it is common practice to express the Nusselt number, Nu_∞, for fully developed conditions as a product of $(\mathrm{Nu}_\infty)_{c.t.}$ for a circular tube and a correction factor, ψ, or

$$\mathrm{Nu}_\infty = \psi * \left(\mathrm{Nu}_\infty\right)_{c.t.} \qquad \text{(for flow along tube bundles)} \tag{12}$$

where $(Nu_\infty)_{c.t.}$ is given by eqn. (11) for the conditions outside the tube bundle. There are a variety of correlations for ψ for different bundle geometries.

These equations may be used for moderate temperature differences, $T_s - T_\infty$, with all properties evaluated at the bulk fluid temperature, T_∞. Here T_s refers to the medal surface temperature (T_i or T_o in the geometry of interest here).

Sieder-Tate Correlation

A popular alternate correlation that is used when the flow is characterized by larger property variations is the Sieder-Tate empirical correlation,

$$Nu_\infty = 0.027\,Re^{0.8}\,Pr^{1/3}\left(\frac{\mu}{\mu_s}\right) \tag{13}$$

where all the properties, except μ_s, are evaluated at the bulk fluid temperature. The viscosity at the surface, μ_s, is evaluated at the surface temperature, T_s (either T_i or T_o, respectively, for the inside or outside surface of the steam generator tube).

Even though eqn. (13) is rather straightforward, the impact of using the Sieder-Tate formulation [eqn. (13)] instead of he Dittus-Boelter correlation [eqns. (10) and (11)] is significant, since it makes the system a nonlinear BVP -- because of the dependence of h_p on T_i and h_s on T_o. Thus, the Dittus-Boelter equation is usually selected, not because of its accuracy relative to other correlations, but because the Sieder-Tate correlation introduces a nonlinearity into the system -- which then often requires an iterative solution scheme.

Finally, given information on the fluid environments, eqns. (6) - (13) can be evaluated to get the Nusselt number on both the primary and secondary sides of the steam generator. For the Dittus-Boelter correlation, one can simply evaluate these expressions for the desired heat transfer coefficients, h_p and h_s, which allows a quantitative evaluation of the temperature distribution in the tube metal via eqns. (2) - (4). If however, the Sieder-Tate correlation is used, one must first guess the surface temperatures, compute h_p and h_s, and solve for the T(r) distribution, which gives new T_i and T_o surface temperatures -- which are then used to compute new values of h_p and h_s, etc. etc.. This iterative process continues until the surface temperatures converge to within some desired tolerance.

Sample data for a specific situation are given below so that one can actually implement and simulate the models described here.

Problem-Specific Data:

Geometry/Material Data:

inside diameter of tube = 7/8 in.

outside diameter of tube = 1 in.

tube material is Inconel with k_m = 35 W/m-°C

lower shell shroud inside diameter = 1.8 m

total number of tubes = 3800

Primary Side:

flow rate = $\dot{m}_p$ = 1.184 kg/s per tube

bulk water temperature = T_p = 305 °C

for saturated water at 305 °C

ρ = 701.6 kg/m^3 k = 0.538 W/m-°C c_p = 5906 J/kg-°C

Secondary Side:

flow rate = $\dot{m}_s$ = 480 kg/s

bulk water temperature = T_s = 280 °C

for saturated water at 280 °C

ρ = 750.3 kg/m^3 k = 0.581 W/m-°C c_p = 5289 J/kg-°C

correction factor for Nusselt number outside tube bundle = ψ = 0.98

Viscosity versus Temperature (for 270 C ≤ T ≤ 320 C):

$$\mu(T) = 2.0\times10^{-4} - 3.801\times10^{-7}\,T$$

where T is in ^{o}C and μ has units of Pa-s.

Reference: The basic idea for this development was obtained from ***Nuclear Systems I*** by Todreas and Kazimi (Hemisphere Publishing Corp., 1990). The problem description given here represents a merger of several sample problems in the chapter on single-phase heat transfer.