10.303 & 14.301 Fluid Mechanics

Extra Credit Project #1 Fall 2006

Tank Flow Dynamics

Project Overview

The continuity equation for a control volume (CV) was developed in class and is given as

$$\frac{d}{dt} m_{CV} = \dot{m}_{in} - \dot{m}_{out} \tag{1}$$

where m_{CV} is the fluid mass within the CV and $\dot{m}_{in}$ and $\dot{m}_{out}$ are the mass flow rates into and out of the control volume, respectively.

This project is designed to give you further experience using this mass balance equation within a variety of related, but different, circumstances. In particular, we look at the flow dynamics of a single tank where the flow rate out of the tank, $\dot{m}_{out} = \rho v_e A_e$, can be determined by the use of the Bernoulli equation applied from the top free surface of the fluid within the tank (denoted with subscript t) to the tank exit (denoted with subscript e), which is assumed to be open to the atmosphere, or

$$\frac{P_t}{\gamma} + \frac{v_t^2}{2g} + z_t = \frac{P_e}{\gamma} + \frac{v_e^2}{2g} + z_e \tag{2}$$

where $h = z_t - z_e$ is the height of the fluid in the tank. Note that the application of this expression assumes that friction losses are negligible and that, at each instant in time, a "quasi" steady flow approximation is appropriate.

For each case as described below, you should formally set up the defining equations and solve for the specific quantify of interest. In some cases, the solution of the resultant initial value problem (IVP) given in eqn. (1) can be obtained analytically. However, in most situations, this is not easy to do or it may not even be possible, and one must resort to the use of numerical methods to solve the given IVP. Thus, since a numerical approach is needed in some cases, you are strongly encouraged to use this approach to complete the full project. This subject was discussed in some detail in your Differential Equations class, and here at UMass-Lowell, we emphasize the use of the ***ode23*** or ***ode45*** routines within Matlab to solve problems of this type. Although you are encouraged to use Matlab for this project, any appropriate method/software is acceptable -- as long as you use it properly to solve the situations described below. Independent of the specific software package, be sure to describe the overall methodology and to include a program listing and/or sufficient information for the reader to fully evaluate your solution methodology. Also note that all the plots requested must be computer generated and formally integrated within your final report!!!

Problem Statement

Case 1: Inlet flow rate of 30 gpm with tank top open to atmosphere and h_o = 1.5 ft

Consider a 5 ft high vertical cylindrical tank with a diameter of 3 ft. The tank initially contains 1.5 ft of water. At t = 0 a valve is fully opened in the inlet line to supply a constant flow rate of 30 gpm (gallons per minute) to the tank. At the same time, a valve in the 1 inch diameter exit line at the bottom of the tank is fully opened, and the water in the tank is allowed to flow out to the environment. The top of the tank is vented to the atmosphere.

Based on this description, you should compute and plot the fluid height, h(t), versus time and the exit flow rate, $Q_e(t)$, versus time, and determine the steady state fluid height, h_∞, within the tank. Fully explain the theoretical development, your solution technique, and the overall dynamics that ensues. Also state any assumptions used in your analysis -- and please be careful with units!!!

Case 2: No inlet flow with tank top open to atmosphere and $h_o = h_\infty$ for Case 1

After 60 minutes of operation as described above for Case 1, the inflow valve to the tank is closed with the top of the tank still vented to the atmosphere.

Under this scenario, how long does it take to drain the tank to a height of 1 ft of water? Also compute and plot the full behavior of h(t) and $Q_e(t)$ versus time. Do these plots make sense physically?

Case 3: No inlet flow with tank top closed and $h_o = h_\infty$ for Case 1

After 60 minutes of operation as described above for Case 1, the inflow valve to the tank is closed and the top vent to the atmosphere is also shut off. This gives a closed tank with a fixed mass of air above the fluid. The pressure in this space then obeys the perfect gas law, $P_t V_a = m_a RT$, where V_a is the air volume, m_a is the air mass, etc., and P_t is the absolute pressure at the top of the tank. Since the air volume expands nearly isothermally as the fluid volume decreases, $P_t V_a$ is approximately constant. Thus, for a fixed tank volume, V_{tank}, and an initial fluid volume, $V_o = V(h_o)$, the pressure at the top of the tank versus the fluid height can be written as

$$P_t(h)V_a(h) = P_{to}V_{ao} \qquad \text{or} \qquad P_t(h)\left[V_{tank} - V(h)\right] = P_{to}\left[V_{tank} - V(h_o)\right]$$

or $$P_t(h) = \frac{V_{tank} - V(h_o)}{V_{tank} - V(h)} P_{to} \tag{4}$$

where V(h) is the fluid volume versus fluid height and $P_{to} = P_{atm}$ is the initial pressure at the top of the tank when the vent valve is closed.

Under this new scenario, re-develop the mathematical model for this situation (as appropriate) and simulate its behavior. Plot h(t) and $Q_e(t)$ versus time as before. For this case, does h(t) ever reach 1 ft? If not, why? In general, explain physically what is happening here and use the math model and your simulation to support your explanation.

Documentation

Formal documentation for this problem should include a brief (but complete) description of the problem being solved, the development of the mathematical models, an overview of the solution technique used, and the key results of your analyses including the requested plots, quantitative results, and comparisons. A professional report and a thorough discussion of the results is required for full credit!

This Extra Credit Project will be worth up to 30 extra points towards your HW grade. Partial credit will be given for partially completed work, but only if significant progress towards a complete solution has been made. Team efforts with 2-3 students per team are encouraged for this project! Only one project report is required for each team. A professional job is expected here!!!