

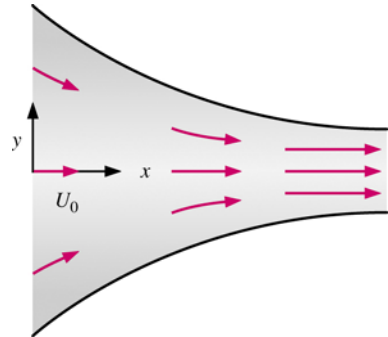
10.303 & 14.301 Fluid Mechanics
Homework Assignment #5 Fall 2006

General Fluid Motion, Control Volume Analysis, and the Continuity Equation

Consider steady, incompressible 2-D flow through a converging duct as shown in the sketch. An approximate velocity field for this flow is given by

$$\vec{v} = (U_0 + bx)\hat{i} - by\hat{j}$$

where U_0 is the horizontal speed at $x = 0$. Note that this equation ignores viscous effects along the walls, but it is a reasonable approximation throughout the majority of the flow field.



This system will be used for Problems 1 – 3.

1. Compute the acceleration field for fluid particles passing through the duct described above.
2. The pressure field in the above duct is given by

$$P = P_0 - \frac{\rho}{2} \left[2U_0bx + b^2(x^2 + y^2) \right]$$

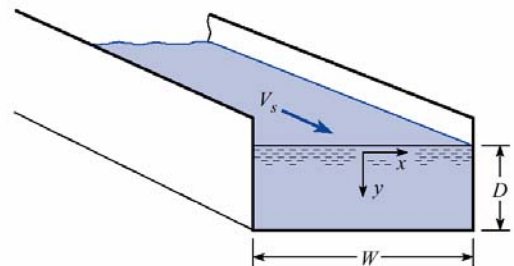
with P_0 as the pressure at $x = 0$. Generate an expression for the rate of change of pressure following a fluid particle within the duct.

3. For the above converging duct flow situation, generate an analytical expression for the flow streamlines. For the case where $U_0 = 5 \text{ ft/s}$ and $b = 4.6 \text{ s}^{-1}$, plot several streamlines from $x = 0 \text{ ft}$ to 5 ft and $y = -3 \text{ ft}$ to 3 ft . Do these streamlines make physical sense for the current situation?

4. Water flows in a 2-D channel of width W and depth D as shown in the sketch. The approximate velocity profile for the water is

$$V(x, y) = V_s \left(1 - \frac{4x^2}{W^2} \right) \left(1 - \frac{y^2}{D^2} \right)$$

where V_s is the velocity of the water at the surface midway between the channel walls. The coordinate system is as shown, with x measured relative to the center plane of the channel and y measured downward from the water surface.

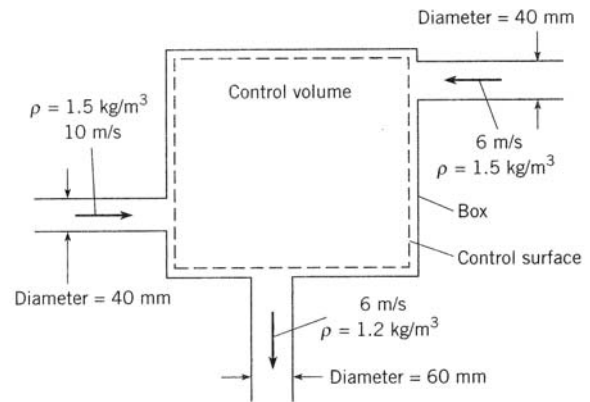


- a. Find the discharge (i.e. volume flow rate) in terms of V_s , D , and W .
- b. What is the average velocity of water in the channel?

5. The Reynold's Transport Theorem was developed in class with the following result:

$$\frac{d}{dt} B_{\text{sys}} = \frac{d}{dt} \int_{\text{CV}} b \rho dV + \int_{\text{CS}} b \rho \vec{v} \cdot \hat{n} dA$$

Within the context of the continuity equation (i.e. the mass balance equation) applied to the flow situation shown in the diagram, answer the following questions:



- What is the numerical value for b for this problem? What does this mean physically?
- Determine a numerical value for the term $\frac{d}{dt} B_{\text{sys}}$. Explain your logic.
- Determine the numerical value of the last term in the equation, $\int_{\text{CS}} b \rho \vec{v} \cdot \hat{n} dA$. Physically what does this term represent? Be explicit here.
- Determine the numerical value of $\frac{d}{dt} \int_{\text{CV}} b \rho dV$. Again, what does this term mean within the context of this problem? Explain your logic...