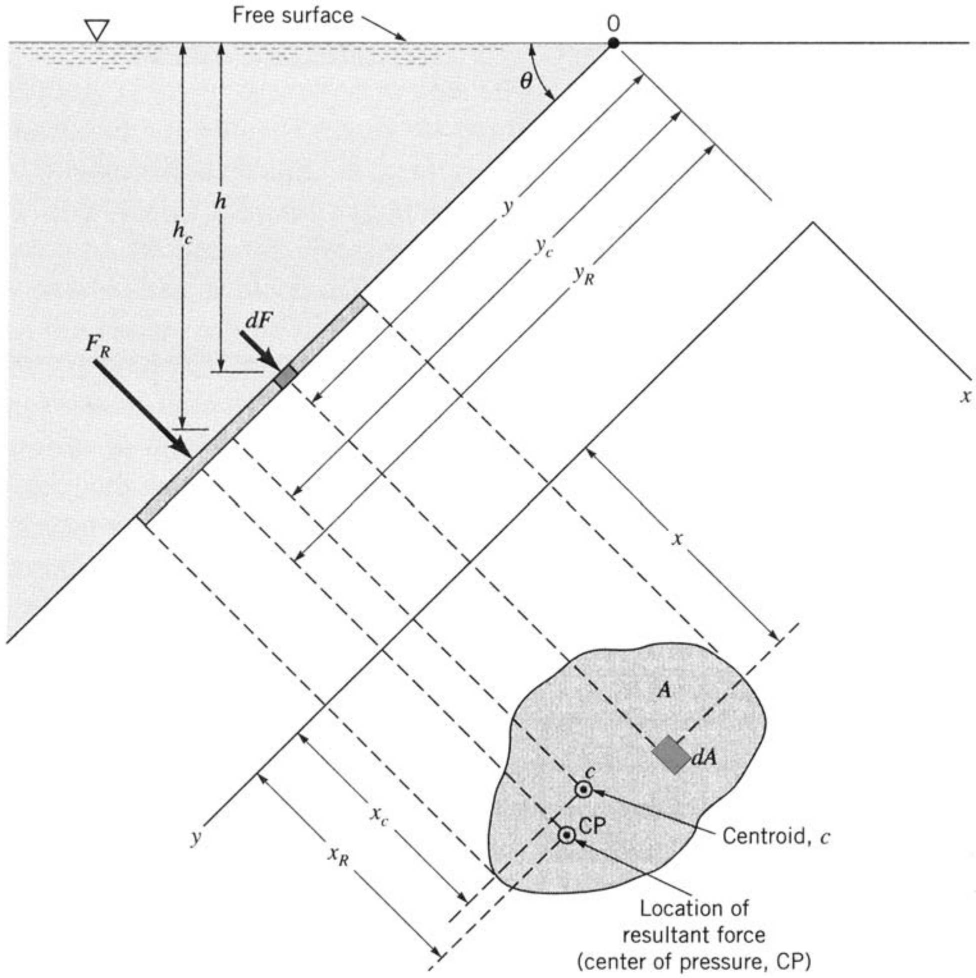
Fluid Mechanics (10.303 or 14.301)

Forces on Plane Surfaces -- Basic Notation



	Munson	**Mott**	**Crowe**	**White**
depth of fluid to differential area dA	h	h	$y \sin\alpha$	h
depth to centroid of area	h_c	d_c	$\bar{y} \sin\alpha$	h_{cg}
distance from fluid surface along solid surface to area dA	y	y	y	ξ
y distance to centroid of area	y_c	L	$\bar{y}$	0
y distance to center of pressure	y_R	L_p	y_{cp}	y_{cp}
distance along solid surface area from x_{ref}	x	---	x	---
x distance to centroid of area	x_c	---	$\bar{x}$	0
x distance to center of pressure of area	x_R	---	$\bar{x}$	x_{cp}
location of resultant force, F_R, is called the center-of-pressure	(x_R, y_R)	$(0, L_p)$	$(\bar{x}, y_{cp})$	(x_{cp}, y_{cp})

Notes: In Mott, the x-axis is not treated explicitly since it is assumed that $x_c = x_R = x_{ref} = 0$. Crowe uses α instead of θ as the angle between the free surface and the inclined planar surface. Crowe also assumes symmetry in the x-direction, which leads to $x_{cp} = \bar{x}$. In White, $x_c = x_{ref} = 0$, and y is also measured relative to the centroid. Thus, the x_{cp} and y_{cp} values are relative to the centroid location.

Summary Equations (using the notation from Munson)

Force Magnitude

$$F_R = \int P\,dA = \int \gamma h\,dA = \gamma \sin\theta \int y\,dA$$

$$\boxed{F_R = \gamma h_c A}$$

where $P = \gamma h$ and $h = y\sin\theta$

and $y_c = \dfrac{\int y\,dA}{\int dA} = \dfrac{1}{A}\int y\,dA$ and $x_c = \dfrac{\int x\,dA}{\int dA} = \dfrac{1}{A}\int x\,dA$ $\quad (x_c, y_c) \equiv$ centroid of area

Thus, $F_R = \gamma \sin\theta\, y_c A = \gamma h_c A$

Center of Pressure

$$F_R y_R = \int y P\,dA = \gamma \int y h\,dA = \gamma \sin\theta \int y^2\,dA$$

Therefore $\quad y_R = \dfrac{\int y^2\,dA}{y_c A} = \dfrac{I_x}{y_c A} \quad$ where $I_x \equiv$ moment of inertia

$$F_R x_R = \int x P\,dA = \gamma \int x h\,dA = \gamma \sin\theta \int xy\,dA$$

Therefore $\quad x_R = \dfrac{\int xy\,dA}{y_c A} = \dfrac{I_{xy}}{y_c A} \quad$ where $I_{xy} \equiv$ product of inertia

But the Parallel Axis Theorem tells us that

$I_x = I_{xc} + A y_c^2 \quad$ where $I_{xc} \equiv$ moment of inertia about the centroid

$I_{xy} = I_{xyc} + A x_c y_c \quad$ where $I_{xyc} \equiv$ product of inertia about the centroid

Thus,

$$\boxed{y_R = y_c + \frac{I_{xc}}{y_c A}} \qquad \boxed{x_R = x_c + \frac{I_{xyc}}{y_c A}}$$

where I_{xc} and I_{xyc} can be found in appropriate references for many common planar geometries.

Treating Composite Areas

$$A = \sum_i A_i \qquad F_R = \sum_i F_{R_i}$$

$$y_c = \frac{1}{A}\sum_i y_{c_i} A_i \qquad x_c = \frac{1}{A}\sum_i x_{c_i} A_i$$

$$y_R = \frac{1}{F_R}\sum_i F_{R_i} y_{R_i} \qquad x_R = \frac{1}{F_R}\sum_i F_{R_i} x_{R_i}$$