

Differential Equations (92.236)

Project #1 Spring 2007

Energy Balance Calculations for Heat-Up of the UMLRR Pool

Background

UMass-Lowell has a 1 MW nuclear reactor (UMLRR) on campus that is used for training and a variety of research activities. The reactor core, where the energy is generated from the nuclear fission process, is quite small. The core itself is located near the bottom of a large pool of water (about 76000 gallons), contained within thick reinforced concrete walls. The large pool of water provides a heat sink for the energy generated within the core, and both the water and concrete walls provide radiation shielding so that no neutron or gamma radiation can escape. The pool surface is open to the atmosphere inside of containment, where energy exchange via evaporation, convection, and radiation heat transfer can occur (the 25 ft of water above the core acts as radiation shielding in this direction).

The reactor can operate in either forced or natural convection mode. With forced convection, the primary coolant system normally removes energy from the core (and pool) through a heat exchanger to the secondary-side cooling tower, which eventually releases the waste energy to the outside atmosphere. The primary coolant flow rate in this mode is about 1650 gpm. In natural convection mode, energy from the core simply heats up the pool water, which is eventually dissipated through the pool surface to the containment atmosphere.

A simple conceptual sketch of the energy flows in the large UMLRR pool is shown in Fig. 1. This shows that energy can enter the pool via flow into the pool and via the internal energy generation associated with the fission process in the reactor core. Energy loss paths include the

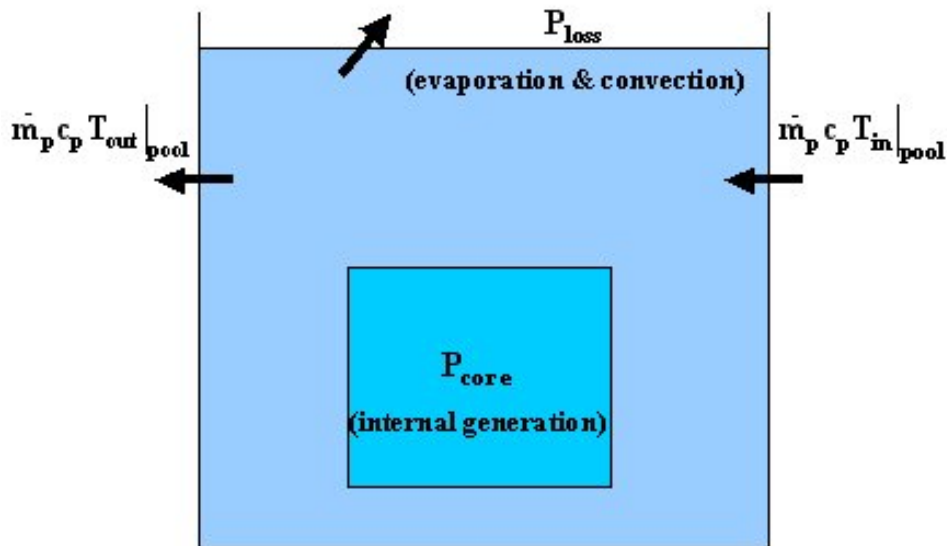


Fig. 1 Sketch showing the key energy transport paths in the UMLRR pool.

energy that is transported with the coolant leaving the system and via several heat transfer mechanisms (evaporation, convection, and radiation) at the pool surface. Note that, over relatively short periods of time, only negligible amounts of energy loss occurs through the thick concrete pool walls (so this is not shown in Fig. 1).

With the energy generation and loss paths shown in Fig. 1, we can write an energy balance for the pool water, as follows:

$$\text{rate of change of energy in pool} = \text{net rate of energy flow into the pool} + \text{internal generation rate of energy within the pool} - \text{rate of energy loss at surface of the pool} \quad (1)$$

If we assume that the evaporation losses have a negligible effect on the mass of water in the pool, and that the mass flow rates in and out of the pool are in balance, then the pool mass is essentially constant. In this case, the terms in the energy balance equation are:

$$\text{rate of change of energy in pool} = \frac{d}{dt} E_{\text{pool}} = \frac{d}{dt} (mc_v T_{\text{pool}}) = mc_v \frac{d}{dt} T_{\text{pool}}$$

$$\text{net rate of energy flow into the pool} = \left(\dot{m}_p c_p T \right) \Big|_{\text{in}} - \left(\dot{m}_p c_p T \right) \Big|_{\text{out}} = \dot{m}_p c_p (T_{\text{in}} - T_{\text{out}}) \Big|_{\text{pool}}$$

$$\text{internal generation rate of energy within the pool} = P_{\text{core}} \quad (\text{where } P_{\text{core}} \text{ is the operating power level of the core})$$

$$\text{rate of energy loss at surface of the pool} = hA (T_{\text{pool}} - T_{\text{air}}) \quad (\text{where } h \text{ is an effective heat transfer coefficient})$$

Now, putting these terms into eqn. (1), with $c = c_v = c_p$ for incompressible fluids, gives

$$mc \frac{dT_{\text{pool}}}{dt} = \dot{m}_p c (T_{\text{in}} - T_{\text{out}}) \Big|_{\text{pool}} + P_{\text{core}} - hA (T_{\text{pool}} - T_{\text{air}})$$

$$\text{or} \quad \frac{dT_{\text{pool}}}{dt} = \frac{\dot{m}_p}{m} (T_{\text{in}} - T_{\text{out}}) \Big|_{\text{pool}} + \frac{1}{mc} P_{\text{core}} - \frac{hA}{mc} (T_{\text{pool}} - T_{\text{air}}) \quad (2)$$

There are a number of approximations built into eqn. (2). Probably the most important consideration is that we have ignored any spatial dependence in the above development. Clearly the water temperature near the heat source (i.e. the reactor core) will be larger than the average pool temperature. In addition, the pool is physically quite large and it has rather complex internal flow patterns that differ significantly for the natural convection and forced convection modes. Thus, by characterizing the pool energy content by a single average pool temperature, T_{pool} , we are ignoring all this detail. However, even with the simplifications made here, we should be able to get a pretty good idea of how various factors affect the overall energy balance in this system.

Now, for the current project, we want to analyze this system under a specific set of conditions that lead to the *Heat-Up of the UMLRR Pool*. In particular, on Feb. 27, 2007, the reactor was started up at about 11 am and operated at about 93% of rated power (i.e. $P_{\text{core}} \approx 0.93 \text{ MW}$) for several hours at constant power conditions. During this time, the secondary side energy removal system was turned off (i.e. the secondary pump and cooling fans were off for most of the reactor run). Under this scenario -- with energy generation in the core, but minimal energy removal -- we expect all the temperatures on the primary side to increase.

Selected data from the actual experiment are displayed in Fig. 2. Here you can see that, shortly after the system is brought to power (at about 20 minutes on the plot), all the temperatures appear to increase in a nearly linear fashion until the cooling pump and fans are turned on just before 5.5 hours of elapsed time. One can see that the secondary is indeed off for the bulk of the period shown, for example, by noticing that the pool inlet and outlet temperatures are nearly the same, indicating that negligible energy is removed from the primary coolant loop via the heat exchanger (secondary pump is off during this time period). Also, note that, with the average pool temperature in the range of 68 F – 72 F (the next to bottom curve in Fig. 2) during the first hour or so of full power operation, and the containment air temperature near 70 F during the whole experiment, we would not expect much energy loss through the pool surface during this time interval. However, at later times, especially when the pool temperature is in the 80 – 90 F range, the difference between the pool temperature and the air temperature may be sufficient to allow some measurable energy loss from the surface of the pool. In fact, addressing the topic of energy loss at the pool surface is one of the key goals of this project (see below).

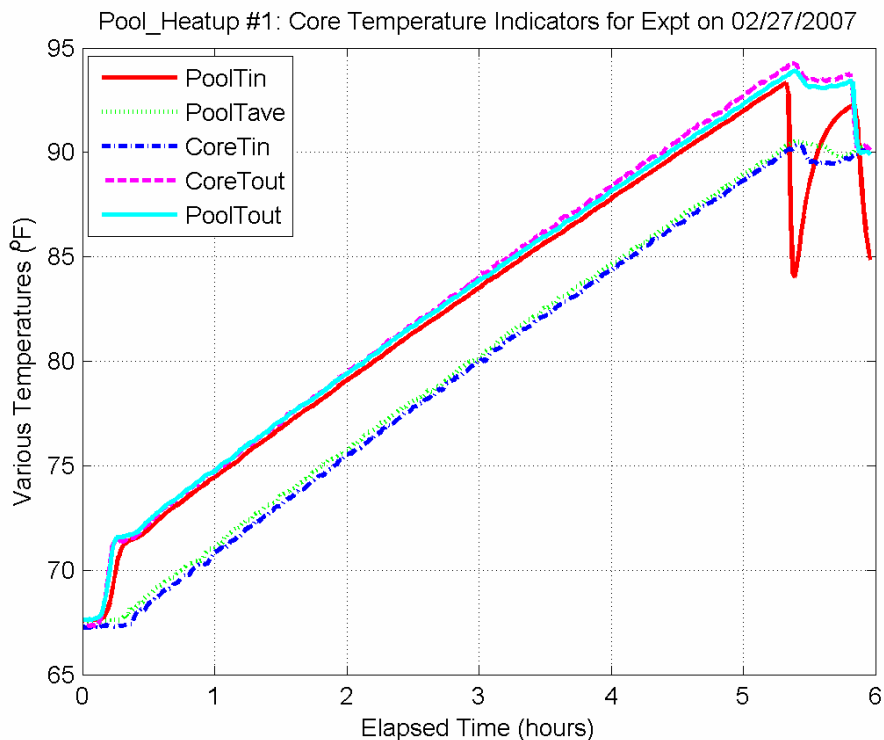


Fig. 2 Selected data for a portion of the UMLRR experiment on February 27, 2007.

Problem Description

Part A -- No losses from pool surface or from the primary side

For the first part of your analysis, assume that the energy loss from the pool surface is small (that is, $hA(T_{\text{pool}} - T_{\text{air}}) \rightarrow 0$) and that there is no energy loss from the primary system (that is, $T_{\text{in}} \approx T_{\text{out}}$ for the pool). Under these assumptions eqn. (2) becomes

$$\left. \frac{dT_{\text{pool}}}{dt} \right|_{\text{no losses}} = \frac{1}{mc} P_{\text{core}} \quad \text{with } T_{\text{pool}}(0) = T_o \quad (3)$$

Solve this initial value problem (IVP) using the data in Table 1 to compute any needed constants for this situation. You should evaluate the pool heat-up rate (dT_{pool}/dt) and the pool temperature profile versus time. Do these results compare with the actual experimental data observed during the first part of the experiment as shown in Fig. 2? For this analysis, you should only use the first 60 minutes of the available data after the reactor has reached constant power. Explain your results fully...

Note: For ease in comparing your analytical temperature profile to the measured data, a short Matlab script file, **tave_expt_022707.m**, containing the experimental data is available on the course website. You should use these data for evaluating the adequacy of all the mathematical models developed as part of this project.

Part B -- No losses from pool surface

As seen in Fig. 2, there is actually a small, relatively constant difference between the pool outlet and inlet temperatures, which suggests that there are indeed some small losses from the primary side piping system. However, the observed temperature difference is so small (only about 0.1 to 0.3 F) that it cannot be measured accurately, especially considering that the measured temperature differences are only accurate to within about 0.1 to 0.2 F at the time of calibration.

However, one way to estimate the pool heat-up including these losses is to fit a linear polynomial directly to the measured data during the first hour of constant power operation (while the pool surface losses are still negligible). During this time interval, the appropriate pool energy balance equation is given by

$$\left. \frac{dT_{\text{pool}}}{dt} \right|_{\text{no surface losses}} = \left. \frac{dT_{\text{pool}}}{dt} \right|_{\text{net}} = \frac{\dot{m}_p}{m} (T_{\text{in}} - T_{\text{out}}) \Big|_{\text{pool}} + \frac{1}{mc} P_{\text{core}} \quad (4)$$

Thus, the slope of the temperature versus time data during this time period is just the net pool heat-up rate and, since both the power level and the primary side losses are constant, the term represented by eqn. (4) is also constant throughout most of the experiment.

In Part A you computed a heat-up rate based only on the power generated in the core assuming no losses. Here you should do the same analysis using the net heat-up from Table 1 (obtained from a linear curve fit) that accounts for the small, but apparent losses from the primary side piping. Add this new profile to the plot generated in Part A, being sure to properly label all three curves. Does this new profile agree better with the experimental data? Does the difference between the Part A and Part B profiles make sense to you? Explain...

Part C -- Include all the important losses

Beyond the first hour or so of operation, the pool temperature starts to get large compared to the air temperature inside containment. As this temperature difference grows, the energy loss from the pool surface can no longer be ignored. Thus, during this portion of the experiment the full balance relationship given in eqn. (2) is required. However, because the power level and primary side losses are constant during this experiment, we can write eqn. (2) as

$$\left. \frac{dT_{\text{pool}}}{dt} \right|_{\text{actual}} = \left. \frac{dT_{\text{pool}}}{dt} \right|_{\text{net}} - \frac{hA}{mc} (T_{\text{pool}} - T_{\text{air}}) \quad \text{with } T_{\text{pool}}(0) = T_o \quad (5)$$

where the net pool heat-up rate is a constant (with the value given in Table 1). This IVP says that the pool temperature will increase because of the energy generated by the reactor core minus the primary side losses, and that it can decrease because of losses at the pool surface. Since the surface losses are generally expected to be relatively small, we expect that the pool temperature will still tend to increase even with this term included -- thus, the emphasis on the *Heat-Up of the UMLRR Pool*.

Therefore, for this part of the analysis, you should not ignore the surface loss term. In particular, you should solve the full IVP given in eqn. (5) using, as before, the data in Table 1 to evaluate the equation constants (be careful with units here). Evaluate and plot the resultant temperature profile assuming that the reactor is run in this mode for a full 5 hours of elapsed time (this is the range given in the **tave_expt_022707.m** data file). For comparison purposes, also plot the result of Part B over this same time period (put the Part B and Part C results on the same curve along with the experimental data).

Now, the only issue here is the heat transfer coefficient, h , since there is no numerical value for this quantity in Table 1. In practice, h is quite sensitive to the environmental conditions, and it can vary over a relatively wide range. In addition, it is really a time-dependent parameter, since the pool and containment air conditions vary with time. However, the actual computation of h from fundamental relationships is quite complicated and it can also complicate the overall mathematical model significantly (and all this is well beyond the scope of this project). Thus, to simplify the analysis here, we will treat h as a constant and perform a parametric study to evaluate how well the computed temperature profile for a given value of h compares to the measured data.

In particular, your job here is to solve the IVP in eqn. (5) and to find the value of h that gives the best fit to the observed data (here we define ‘best’ as the value of h that gives a good visual comparison). A rough hand calculation for h using some correlations from the literature gave a first guess for the effective heat transfer coefficient for the current analyses as $h_{\text{nom}} = 15 \text{ BTU/hr-ft}^2\text{-F}$. However, in practice, the uncertainty here could easily give a real value that is higher or lower than this rough estimate by as much as a factor of five. Thus, you should program the solution to the IVP and compare the resultant $T_{\text{pool}}(t)$ profile to the measured data for a value of h that is interactively input into your simulation program by the user. Then, by simply running your program a number of times, you should be able to narrow in on the actual value of the effective heat transfer coefficient that is appropriate for this problem -- this is the primary goal of the Part C analysis.

Also, as usual, make sure you thoroughly discuss your solution procedure and the results of your analysis! For example, do the two models, with and without surface losses, give similar behavior during the first hour or so? What about operation for a longer period of time? Also, what happens as the heat transfer coefficient increases? Try to explain what is happening here...

Part D -- Compute time to reach 100 F

As a final evaluation, we note that the UMLRR has a Technical Specification that limits the pool temperature to about 108 F and, to assure that this is never exceeded, the practical operating

limits are set below this value. For example, let's choose 100 F as the practical upper limit for the average pool temperature. Based on this condition, and the Case B and Case C models analyzed above, estimate how long the reactor could run under the current conditions, if the initial pool temperature is 67.9 F. Does including the surface energy loss term make a large difference in this estimate? Explain...

Documentation

Documentation for this problem should include a complete description of the problem to be solved, the details of the solution techniques, and the key results of your analyses including the requested plots and comparisons. Try to analyze and discuss the results in as much detail as possible. Also include, in an appendix, any program listings (or tabular data if using a spreadsheet) used in generating the summary plots.

This is a formal project and a professional report documenting your work is expected. Your report should have a brief **Introduction** that overviews the problem of interest, a **Procedure** section that discusses the problem formulation, the solution procedure, and any required derivations, and a **Results** section that summarizes the key results in tabular and/or graphical form, as appropriate. Be sure to thoroughly explain the logic used in your analysis and to fully discuss the results obtained.

This Project will be worth 50 points towards your HW grade in this course. Partial credit will be given for partially completed work, but only if significant progress towards a complete solution has been made and a professional report is submitted. Team efforts with 3-4 students per team are encouraged for this project! Each team should only pass in a single, professional project report that is signed and dated by each team member, signifying that they contributed in a meaningful way to the final product. Each member of the team will receive the same grade.

Good luck and have fun -- this should be an interesting project!!!

Table 1 Numerical data for use in project calculations.

Water Properties:	Pool Properties (approx.):
density = 62.2 lbm/ft ³	volume = 76000 gal
specific heat = 1.0 BTU/lbm-F	surface area = 500 ft ²
Operational Parameters (secondary off):	Conversion Factors:
approximate core power = 0.93 MW	1 ft ³ = 7.48 gal
initial pool temperature = 67.9 F	1 MW = 1000 kW
containment air temperature = 70.3 F	1 kW = 3412.14 BTU/hr
primary coolant flow rate = 1650 gpm	1 hr = 60 min
net heatup rate with primary side losses = 0.08 F/min	
Note: For consistency, you should let time be in minutes and, in general, be very careful to use consistent units throughout your calculations.	