

## Differential Equations (92.236)

### Homework Assignment #8 Spring 2007

#### *Numerical Approximations: Improved Euler and Runge Kutta Methods*

##### Problem 1

Consider the following IVP:

$$\frac{dy}{dx} = \frac{2e^{2x} - y}{x} \quad \text{with} \quad y(1) = e^2 \approx 7.389$$

with analytical solution:  $y(x) = e^{2x}/x$

- Using the Improved Euler method with a step size of  $h = 0.5$ , perform a series of hand calculations to estimate  $y(x)$  for  $1 \leq x \leq 2$ . Create a short table of results and compare the Improved Euler solution to the exact result given above.
- Use the **impeuler.m** Matlab function to solve the above problem using step sizes of 0.5, 0.2, and 0.1, again comparing the analytical and numerical solutions. You should modify the **ndemo2.m** sample program discussed in the Matlab lab, as necessary, to perform these calculations (**ndemo2.m** is the main program which calls **impeuler.m** to actually implement the Improved Euler method). Do the Matlab results with  $h = 0.5$  agree with your hand calculations from Part a? With the maximum errors from the two cases with step sizes of 0.2 and 0.1 (two different step sizes) denoted as  $\epsilon_1$  and  $\epsilon_2$ , address how the error in the Improved Euler method varies with the step size,  $h$ . Is this what you expected? Explain...
- Redo Part b (the Matlab calculations only) using the 4<sup>th</sup> order Runge Kutta method (see **ndemo3.m** and **rk4.m**).

##### Problem 2

Do the same computations and analyses as for Prob. #1 for the following IVP:

$$\frac{dy}{dx} = y - x - 1 \quad \text{with} \quad y(0) = 1$$

with analytical solution:  $y(x) = 2 + x - e^x$

- Do hand calculations using the Improved Euler method with  $h = 0.25$  over the range  $0 \leq x \leq 0.5$ .
- Perform Matlab calculations with  $h = 0.25, 0.1$  and  $0.05$  and develop a relationship between error and step size for the Improved Euler method.
- Redo Part b using the RK4 method.

**Note:** Documentation for this HW assignment should include the hand calculations performed using the Improved Euler method, a copy of selective output results and/or plots from the Matlab calculations, and a brief discussion of your observations concerning the numerical error associated with the Improved Euler method and RK4 method. There is a lot of stuff here, so try to keep it as organized as possible...