

Differential Equations (92.236)
Homework Assignment #3 Spring 2007
Separable Equations and Applications

Problem 1

Solve the following separable ODEs:

- a. $\frac{dy}{dx} = 2xy^3$ with $y(0) = 1$
- b. $y^3 dx - x^2 dy = 0$ with $y(2) = 1$
- c. $(xy + y)dx + (xy + x)dy = 0$
- d. $\frac{dy}{dx} = xy + 4y + 3x + 12$
- e. $\frac{du}{dt} = \frac{t^2 + 1}{u^2 + 4}$ with $u(0) = 1$

Problem 2

\$2000 is placed into an account that pays 5% interest compounded continuously. The interest payments are allowed to accumulate. How much will the account contain in 10 years?

Problem 3

According to one cosmological theory, there were equal amounts of two uranium isotopes, U235 and U238, when the universe was created. At present, there are 137.7 atoms of U238 for each atom of U235. Given the half-lives

$$T_{1/2} = 4.51 \times 10^9 \text{ yrs for U238} \quad \text{and} \quad T_{1/2} = 7.10 \times 10^8 \text{ yrs for U235}$$

estimate the age of the universe (assuming that this particular theory is valid).

Problem 4

A cake is removed from an oven at 210 F and is left to cool in a room at 70 F. After 30 minutes the temperature of the cake is 140 F. Estimate when the cake will be at 100 F.

Also make a plot of the cake temperature versus time using Matlab. Put your name within the title of the plot and submit only the resultant plot along with the hand calculations for this problem (the Matlab program listing is not required). Do your Matlab plot and the hand computations agree?