

Differential Equations (92.236)

Homework Assignment #14 Spring 2007

Numerical and Symbolic Solution of Higher Order Systems

Part A: Solve Problems 1-3 (see below) using the following **numerical** solution procedure:

- Convert each 2nd order equation into two 1st order equations.
- Solve the system of 1st order ODEs with Matlab's ODE solver, **ode23**.
- Plot and interpret the solutions and compare these to the analytical result that is given in the problem description.
- Comment on the ease of use of the numerical scheme relative to the analytical solution method and on the accuracy of the solutions.

Note: All three problems should be solved in a single Matlab script file, with three separate ODE function files (of course). Plot the numerical and analytical solutions for $x(t)$ (Prob. 1 & 2) and $I(t)$ (Prob. 3), as appropriate, on the same axes, but create a separate, properly labeled plot for each problem (i.e. you should have three separate plots for Part A.).

Part B: Now solve Problems 1-3 (see below) using Matlab's **symbolic** processing capability. Again all three problems should be solved in a single Matlab script file. Make each solution “**pretty**”, and include a copy of the command window output as part of your documentation for this problem. Do the solutions using **dsolve** agree with the analytical solutions given below? What do you think of the general symbolic capability that is available in the Symbolic Toolbox?

Problem 1: $x'' + 10x' + 125x = 0$ with $x(0) = 6$ m and $v(0) = x'(0) = 50$ m/s

with analytical solution: $x(t) = e^{-5t} (6 \cos 10t + 8 \sin 10t)$

Problem 2: $x'' + 4x' + 5x = 10 \cos 3t$ with $x(0) = 0$ m and $v(0) = x'(0) = 0$ m/s

with analytical solution: $x(t) = \frac{5\sqrt{2}}{4} e^{-2t} \cos(t - 4.8543) + \frac{\sqrt{10}}{4} \cos(3t - 1.8925)$

Problem 3: $I'' + 8I' + 25I = 0$ with $I(0) = 0$ amps and $I'(0) = -75$ amps/s

with analytical solution: $I(t) = -25e^{-4t} \sin 3t$

Documentation:

Along with the brief comments requested above, also include the derivations required to convert each 2nd order equation into two 1st order equations (Part A). Also attach a listing of the Matlab programs and output plots as part of your solutions for this HW assignment.