

## Differential Equations (92.236)

### Homework Assignment #13 Spring 2007

#### *Applications Involving 2<sup>nd</sup> Order Linear Equations*

##### **Problem 1**

A mass of 3 kg is attached to the end of a spring that can be stretched 20 cm by a force of 15 N. If the mass is set in motion with  $x_0 = 0$  m and  $v_0 = -10$  m/s, find the magnitude, period, and frequency of the resulting undamped motion.

##### **Problem 2**

An unforced mechanical system contains a mass, spring, and dashpot (with  $m = 1$  kg,  $k = 125$  N/m, and  $c = 10$  kg/s).

- If the initial position is  $x_0 = 6$  m and the initial velocity of the mass is  $v_0 = 50$  m/s, find the position versus time,  $x(t)$ .
- If  $x(t)$  from Part a is written in the form,  $x(t) = A(t)\cos(\omega t - \alpha)$ , what is the amplitude function  $A(t)$ , the circular frequency  $\omega$ , and the phase angle  $\alpha$  for this situation?

##### **Problem 3**

Consider an automobile that weighs 3200 lbf in a gravity field with  $g = 32$  ft/s<sup>2</sup>.

- In a test environment, the shock absorbers are disconnected from the suspension system. The undamped frequency of oscillation of the vehicle is measured to be 80 cycles/min. With this information, estimate the value of the linear spring constant. What are the units of  $k$ ?
- Now, with the shock absorbers connected, the damped natural frequency is 78 cycles/min. What is the value of the linear friction coefficient? What are the units of  $c$ ?
- Again with the shocks attached, estimate the time it takes for an initial oscillation to reduce to about 1% of its initial value.

##### **Problem 4**

A particular mechanical system that is excited by a sinusoidal forcing function is described by the following IVP:

$$x'' + 4x' + 5x = 10\cos 3t \quad \text{with} \quad x(0) = 0 \quad \text{and} \quad x'(0) = 0$$

Determine both the transient and steady periodic components of the solution for this system.

##### **Problem 5**

Consider a series RLC circuit with the following values for the circuit components, applied voltage, and initial conditions:

$$R = 16 \Omega \quad L = 2 \text{ H} \quad C = 0.02 \text{ F} \quad E(t) = 100 \text{ V} \quad I(0) = 0 \quad I'(0) = -75 \text{ amp/s}$$

For this system, determine the current versus time relationship,  $I(t)$ .