92.236 Engineering Differential Equations Final Exam Spring 2006

Please write all answers and all work in the blue book provided.
PLEASE SHOW ALL WORK! You will not receive full credit if you do not show your work.

Problem 1. (15 pts.)

Solve the following initial value problem: $xy' + 5y = 7x^2$ with $y(2) = 5$.

Problem 2. (15 pts.)

Solve the following initial value problem: $xyy' = x^2 + 3y^2$ with $y(1) = 2$.

Problem 3. (10 points)

Consider the first-order autonomous equation

$$\frac{dx}{dt} = 4x - x^3$$

a. Draw the phase line for the given equation, noting the equilibrium solutions.

b. Sketch the graphs of the five solutions that satisfy the five initial conditions $x(0) = -2$, $x(0) = -1, x(0) = 0, x(0) = 2, x(0) = 3$.

c. If $x(0) = 3$, determine the limiting value of $x(t)$ as t increases.

Problem 4. (10 points)

Find the general solution to each of the following differential equations:

a. $y'' + 4y' + 5y = 0$

b. $y^{(4)} - 9y'' = 0$

Problem 5. (15 points) Solve the following initial value problem:

$$y'' - 2y' = 8\cos(2x) \text{ with } y(0) = 1 \text{ and } y'(0) = 0.$$

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Problem 6. (15 points)

An unforced mechanical system is described by the linear IVP

$$my'' + cy' + ky = 0 \quad \text{with} \quad y(0) = y_0 \text{ and } y'(0) = v_0$$

where y_0 and v_0 are the initial position (m) and velocity (m/s), respectively.

a. The **undamped, unforced** spring-mass system with $m = 2$ kg has a measured undamped natural frequency of oscillation of about 1.125 cycles/second. Estimate the value of the spring constant k. Be sure to include appropriate units along with the numerical value of k.

b. During normal operation, the spring-mass system is placed in a viscous fluid to introduce a large amount of damping into the overall system. If the measured natural response of the composite **overdamped, unforced** spring-mass-fluid system is

$$y(t) = 10e^{-5.35t} - 8e^{-9.35t}$$

with $y(t)$ measured in meters, estimate the damping coefficient associated with the viscous fluid. Show your computations and explain the logic used to arrive at your result. Also be sure to include appropriate units along with the numerical value of c.

Problem 7. (10 points)

a. Find the Laplace transform of the function f given by $f(t) = e^{-t}\cos(2t)$

b. Find the inverse Laplace transform of the function F given by $F(s) = \dfrac{s-6}{s^2 - 3s}$

Problem 8. (10 points)

Use the Laplace Transform to solve the following initial value problem:

$$x'' + 4x' + 13x = 39 \quad \text{with } x(0) = 0 \text{ and } x'(0) = 0.$$

Only solutions using the Laplace Transform method will be accepted.